10.1 Radical Expressions and Functions

Square Roots

If a is a nonnegative real number, the nonnegative number b such that $b^2 = a$, denoted by $b = \sqrt{a}$, is the principal square root of a.

Example1: Simplify each of the following:

a. $\sqrt{100} = 10$; since $10 \cdot 10 = 100$

b. $\sqrt{\frac{9}{16}} = \frac{3}{4}$; since $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

c. $\sqrt{.04} = 0.2$; since $(0.2)(0.2) = 0.04$

d. $-\sqrt{100} = -10$

e. $\sqrt{9+16} = \sqrt{25}$
$= 5$; since $5 \cdot 5 = 25$

f. $\sqrt{36} + \sqrt{9+16}$
$= 6 + \sqrt{25}$; since $6 \cdot 6 = 36$
$= 6 + 5$
$= 11$

What happens when we try to evaluate the square root of a negative number? The square root of a negative number is not a real number.

Square Root Functions

Because each nonnegative real number, x, has precisely one principal square root, $\sqrt{x}$, there is a square root function defined by

$$f(x) = \sqrt{x}.$$

The domain of this function is $[0, \infty)$.

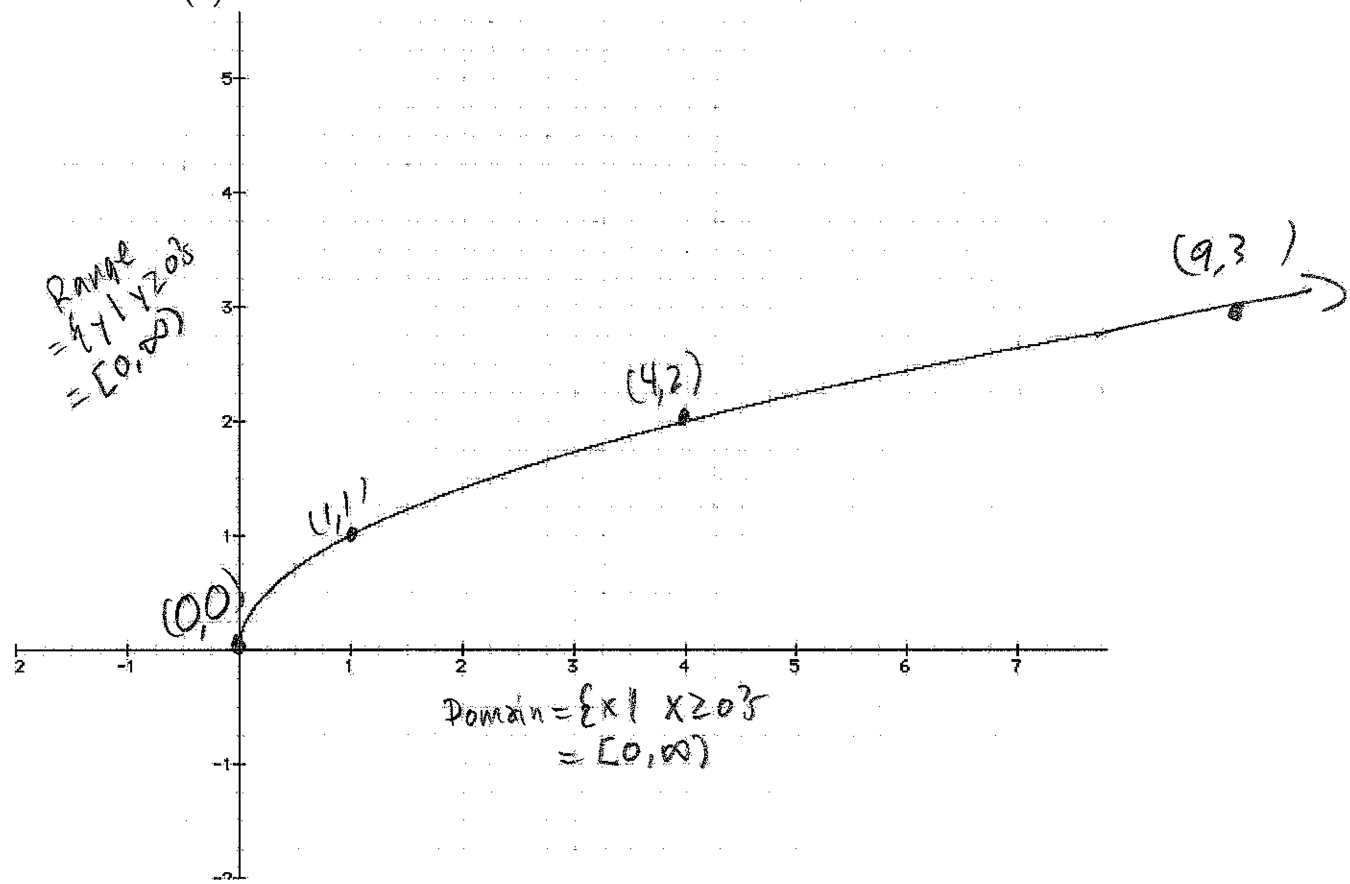
To graph $f(x) = \sqrt{x}$, construct a table of values by choosing nonnegative real numbers for x and calculating y. If you are not using a calculator, it is easiest to calculate y if you choose perfect squares for x. Plot these ordered pairs and then connect the points with a smooth curve. Remember that the domain is **nonnegative** real numbers.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Table of values for $f(x)=\sqrt{x}$

x	$f(x)=\sqrt{x}$	(x,y)
0	$f(0)=\sqrt{0}=0$	(0,0)
1	$f(1)=\sqrt{1}=1$	(1,1)
4	$f(4)=?=\sqrt{4}=2$	(4,2)
9	$f(9)=?=\sqrt{9}=3$	(9,3)
16	$f(16)=\sqrt{16}=4$	(16,4)

Graph of $f(x)=\sqrt{x}$.



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Evaluating Square Root Functions
To evaluate a square root function at a value of x, substitute that value into the function everywhere that x occurs.

Example 2: For each function, find the indicated function value.

a. $f(x)=\sqrt{2x+5}$, $f(2)$ Solution: $f(2)=\sqrt{2(2)+5}=?$
$=\sqrt{4+5}=\sqrt{9}=3$; $\underline{f(2)=3}$

b. $g(x)=\sqrt{x+13}$, $g(3)$ $g(3)=\sqrt{(3)+13}=\sqrt{16}=4$; $\underline{g(3)=4}$

c. $g(x)=\sqrt{x+13}$, $g(-4)$ $g(-4)=\sqrt{(-4)+13}=\sqrt{9}=3$; $\underline{g(-4)=3}$

Finding the Domain of a Square Root Function
Because the square root of a negative number is not a real number, the square root function is defined only for values of the independent variable that produce a nonnegative radicand.

To find the domain of a square root function, set the radicand (quantity under the radical) greater than or equal to zero and solve the inequality.

Example 3: Find the domain of the given square root functions.

a. $f(x)=\sqrt{2x+6}$ Solution: $2x+6\geq 0$
$2x\geq -6$
$x\geq -3$
Domain $=[-3,\infty)$

b. $g(x)=\sqrt{5x-12}$ Solve: $5x-12\geq 0$
$12+5x-12\geq 0+12$
$5x\geq 12$
$\frac{5x}{5}\geq\frac{12}{5}$
$x\geq\frac{12}{5}$
Domain $=\underline{[\frac{12}{5},\infty)}$

c. $h(x)=\sqrt{3-2x}$ Solve:
$3-2x\geq 0$
$-3+3-2x\geq -3+0$
$-2x\geq -3$
$\frac{-2x}{-2}\leq\frac{-3}{-2}$
$x\leq\frac{3}{2}$
Domain $=\underline{(-\infty,\frac{3}{2}]}$

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Modeling With a Square Root Function
The graph of the square root function is increasing from left to right, but the rate of increase is decreasing. Thus, the square root function is often the appropriate function to use to model growth phenomena with decreasing rate of growth.

Example 4: The median height of boys in the United States from birth through age 60 months is given by the function

$$f(x) = 2.9\sqrt{x} + 20.1$$

where x is the age in months and f(x) is the median height. Use this function and a calculator to find the median height of 3-year old boys in the United States. Round your answer to the nearest tenth of an inch.

x= age in months
x = 3 years
x= 36 months

$f(36) = 2.9\sqrt{(36)} + 20.1$
$f(36) = 2.9 \cdot (6) + 20.1$
$f(36) = 17.4 + 20.1$
$f(36) = 37.5$

ANS: The median height of 3 year old boys in the U.S. is approximately 37.5 inches.

Simplifying Expressions of the Form $\sqrt{a^2}$

For any real number a, $\sqrt{a^2} = |a|$.

For any nonnegative real number a, $\sqrt{a^2} = |a| = a$

Example 5: Simplify each of the following:

a. $\sqrt{(-3)^2}$ Solution: $\sqrt{(-3)^2} = |(-3)| = 3$

b. $\sqrt{49x^4} = \sqrt{(7x^2)^2} = |7x^2| = 7x^2$

c. $\sqrt{(x-9)^2} = |x-9|$

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Cube Roots and Cube Root Functions

The cube root of a real number a is written $\sqrt[3]{a}=b$ where $b^3=a$. The cube root of a positive number is positive, and the cube root of a negative number is negative. The following table shows some cube roots of perfect cubes that you should memorize.

Table of Cube Roots of Some Perfect Cubes

$\sqrt[3]{1}=1$	$1\cdot1\cdot1=1$
$\sqrt[3]{8}=2$	$2\cdot2\cdot2=8$
$\sqrt[3]{-8}=-2$	$(-2)\cdot(-2)\cdot(-2)=-8$
$\sqrt[3]{27}=3$	$3\cdot3\cdot3=27$
$\sqrt[3]{-27}=-3$	$(-3)\cdot(-3)\cdot(-3)=-27$
$\sqrt[3]{64}=4$	$4\cdot4\cdot4=64$
$\sqrt[3]{-64}=-4$	$(-4)\cdot(-4)\cdot(-4)=-64$
$\sqrt[3]{125}=5$	$5\cdot5\cdot5=125$
$\sqrt[3]{216}=6$	$6\cdot6\cdot6=216$
$\sqrt[3]{1000}=10$	$10\cdot10\cdot10=1000$

Graph of the Cube Root Function

Because each real number has exactly one cube root, $f(x)=\sqrt[3]{x}$ is a function. It is called the cube root function. The domain of f is the set of all real numbers. To graph the function, construct a table of values by choosing perfect cubes for x and calculating f(x). Then plot each ordered pair and connect the ordered pairs with a smooth curve. A table of values and the graph follow.

Table of values for $f(x)=\sqrt[3]{x}$

x	$f(x)=\sqrt[3]{x}$	(x,y)
-8	$f(-8)=\sqrt[3]{-8}=-2$	$(-8,-2)$
-1	$f(-1)=\sqrt[3]{-1}=-1$	$(-1,-1)$
0	$f(0)=\sqrt[3]{0}=?=0$	$(0,0)$
1	$f(1)=?=\sqrt[3]{1}=1$	$(1,1)$
8	$f(8)=\sqrt[3]{8}=2$	$(8,2)$

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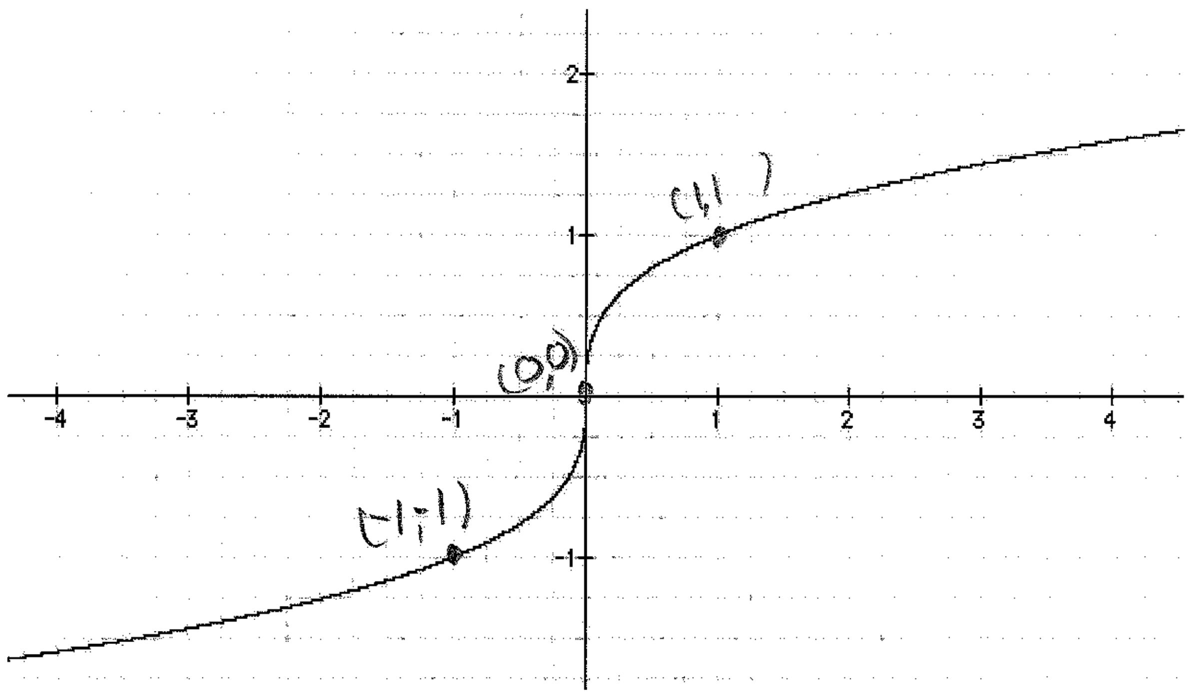

Note that the cube root function is always increasing as the graph is traced from left to right.

Evaluating Cube Root Functions
To evaluate a cube root function at a value of x, substitute that value into the function everywhere that x occurs.

Example 6: Find the indicated function values.

a. $f(x)=\sqrt[3]{x+3}$, $f(24)$ Solution: $f(24)=\sqrt[3]{24+3}=?$

$f(24)=\sqrt[3]{27}$

$f(24)=3$

b. $g(x)=\sqrt[3]{3x-4}$, $g(4)$

$g(4)=\sqrt[3]{3(4)-4}$

$g(4)=\sqrt[3]{12-4}$

$g(4)=\sqrt[3]{8}$

$g(4)=2$

c. $h(x)=\sqrt[3]{1-3x}$, $h(3)$

$h(3)=\sqrt[3]{1-3(3)}$

$h(3)=\sqrt[3]{1-9}$

$h(3)=\sqrt[3]{-8}$

$h(3)=-2$

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Simplifying Cube Roots

For any real number a, $\sqrt[3]{a^3} = a$.

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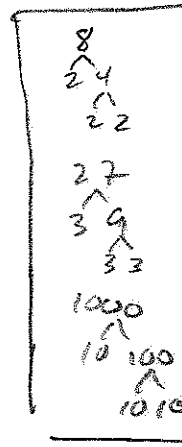

Example 7: Simplify each of the following.

a. $\sqrt[3]{8x^3} = 2x$; since $(2x)\cdot(2x)\cdot(2x) = 8x^3$

b. $\sqrt[3]{-27x^3} = -3x$; since $(-3x)(-3x)(-3x) = -27x^3$

c. $\sqrt[3]{1000x^3} = 10x$; since $(10x)(10x)\cdot(10x) = 1000x^3$

Even and Odd *n*th Roots

Radical expressions can have roots other than square roots and cube roots.

The radical expression $\sqrt[n]{a}$ means the *n*th root of a. The number n is called the index, and a is called the radicand.

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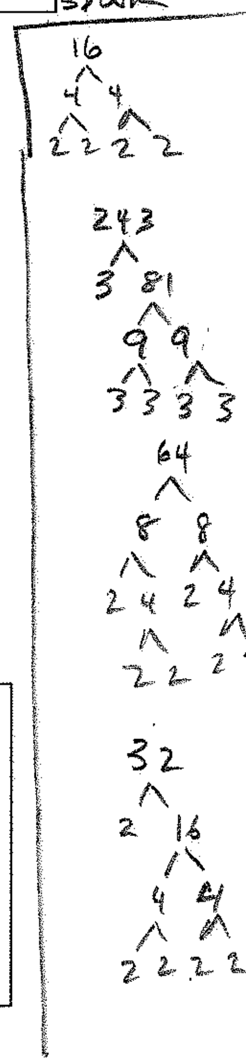

In general, $\sqrt[n]{a} = b$ if $b^n = a$.

Example 8: Simplify each of the following.

a. $\sqrt[4]{16}$ Solution: $\sqrt[4]{16} = 2$; since $2\cdot2\cdot2\cdot2 = 16$

$\sqrt[4]{2^4} = 2$

b. $\sqrt[5]{243} = \sqrt[5]{3^5}$; since $243 = 3\cdot3\cdot3\cdot3\cdot3$

$= 3$

c. $\sqrt[6]{64} = \sqrt[6]{2^6}$; since $64 = 2\cdot2\cdot2\cdot2\cdot2\cdot2$

$= 2$

d. $\sqrt[5]{-32} = \sqrt[5]{(-2)^5}$; since $-32 = (-2)(-2)(-2)(-2)(-2)$

$= -2$

If the index is an odd number, then the root is said to be an odd root. An odd root of a positive number is a positive number, and an odd root of a negative number is a negative number. If the index is an even number, then the root is said to be an even root. Since we choose the principal root when the index is even, the even root of a positive number is a positive number. The even root of a negative number is a not a real number.

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243
3 81
9 9
3 3 3 3

128
2 64
8 8
2 4 2 4
2 2 2 2

Example 9: Find the indicated root, or state that the expression is not a real number.

a. $\sqrt[4]{-16}$, not a real number

b. $\sqrt[5]{-243} = \sqrt[5]{(-3)^5}$
$= -3$

c. $\sqrt[6]{-64}$, not a real number

d. $\sqrt[7]{-128} = \sqrt[7]{(-2)^7}$
$= -2$

Simplifying Expressions of the Form $\sqrt[n]{a^n}$

For any real number a,
If n is even, $\sqrt[n]{a^n} = |a|$
If n is odd, $\sqrt[n]{a^n} = a$

Example 10: Simplify each of the following:

a. $\sqrt[4]{x^4} = |x|$

b. $\sqrt[5]{(x-5)^5} = x-5$

c. $\sqrt[3]{(2x+1)^3} = 2x+1$

d. $\sqrt{(3x-1)^2} = |3x-1|$

Using a Calculator to Find Roots

The square root key on the TI-83 calculator is located on the left side above the x^2 key. Other roots are found in the MATH menu. Press the "MATH" key, and scroll down to $\sqrt[3]{}$ for cube roots or $\sqrt[x]{}$ for roots higher than 3. To use the $\sqrt[x]{}$ function, enter the index first, select $\sqrt[x]{}$, enter the radicand, and then press enter for the desired root.

Example 11: Use your calculator to find the following. Round to three decimal places.

a. $\sqrt{151} \approx 12.288$

b. $\sqrt[3]{58} \approx 3.871$

c. $\sqrt[5]{546} \approx 3.527$

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Answers Section 10.1

Example 1:
a. 10
b. $\frac{3}{4}$
c. 0.2
d. –10
e. 5
f. 11

Example 2:
a. f(2) = 3
b. f(3) = 4
c. g(–4) = 3

Example 3:
a. $[-3,\infty)$
b. $\left[\frac{12}{5},\infty\right)$
c. $\boxed{\left(-\infty,\frac{3}{2}\right]}$

Example 4: x = 36, f(36) = 37.5
The median height of a 3 yr. old boy in the U.S. is 37.5 inches.

Example 5:
a. 3
b. $7x^2$
c. $|x-9|$

Example 6:
a. f(24) = 3
b. g(4) = 2
c. h(3) =–2

Example 7:
a. 2x
b. –3x
c. 10x

Example 8:
a. 2
b. 3
c. 2
d. –2

Example 9:
a. Not a real number
b. –3
c. Not a real number
d. –2

Example 10:
a. $|x|$
b. x – 5
c. 2x + 1
d. $|3x-1|$

Example 11:
a. 12.288
b. 3.871
c. 3.527

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10.2 Rational Exponents

Definition of $a^{\frac{1}{n}}$

If $\sqrt[n]{a}$ represents a real number and $n \geq 2$ is an integer, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}.$$

If n is odd and

- a is positive, then $a^{\frac{1}{n}}$ is positive.
- a is negative, then $a^{\frac{1}{n}}$ is negative.
- a is zero, then $a^{\frac{1}{n}}$ is zero.

If n is even and

- a is positive, then $a^{\frac{1}{n}}$ is positive.
- a is negative, then $a^{\frac{1}{n}}$ is not a real number
- a is zero, then $a^{\frac{1}{n}}$ is also zero.

Example 1: Use radical notation to rewrite each expression. Simplify, if possible.

a. $36^{\frac{1}{2}} = \sqrt{36} = ?$
$= 6$

b. $(-8)^{\frac{1}{3}} = \sqrt[3]{-8}$
$= -2$

c. $(9xy^2)^{\frac{1}{5}} = \sqrt[5]{9xy^2}$

d. $(x^2)^{\frac{1}{2}} = \sqrt[2]{x^2}$
$= |x|$

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Example 2: Rewrite each expression using rational exponents.

a. $\sqrt[4]{5xy} = (5xy)^{\frac{1}{4}}$

b. $\sqrt[3]{3xy^2} = (3xy^2)^{\frac{1}{3}}$

c. $\sqrt[5]{4a^2b} = (4a^2b)^{\frac{1}{5}}$

d. $\sqrt{3xy} = (3xy)^{\frac{1}{2}}$

Definition of $a^{\frac{m}{n}}$

If $\sqrt[n]{a}$ represents a real number and $\frac{m}{n}$ is a positive rational number, $n \geq 2$, then

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m \quad \text{and} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}.$$

Note that if n is even and a is negative, $\sqrt[n]{a}$ does not represent a real number and $a^{\frac{m}{n}}$ is not a real number.

Example 3: Use radical notation to rewrite each of the following and then simplify.

a. $16^{\frac{3}{2}} = \sqrt{16}^3 = 4^3 = ? = 4 \cdot 4 \cdot 4 = 16 \cdot 4 = 64$

b. $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 2 \cdot 2 = 4$

c. $(-9)^{\frac{3}{2}} = \left(\sqrt[2]{-9}\right)^3$; not a real number

d. $-32^{\frac{3}{5}} = -\left(\sqrt[5]{32}\right)^3$
$= -(2)^3$
$= -(8) = -8$

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16 = 4 · 4

$16 = 4^2$

8 = 2 · 4 = 2 · 2 · 2

$8 = 2^3$

32 = 2 · 16 = 2 · 2 · 8 = 2 · 2 · 2 · 4 = 2 · 2 · 2 · 2 · 2

$32 = 2^5$

$2^3 = 2 \cdot 2 \cdot 2$
$= 4 \cdot 2$
$= 8$

Example 4: Rewrite with rational exponents.

a. $\sqrt[4]{8^5} = 8^{5/4}$

b. $\sqrt[3]{(3x)^2} = (3x)^{2/3}$

c. $\left(\sqrt[6]{5xy}\right)^7 = (5xy)^{7/6}$

d. $\sqrt[5]{25x^2} = (25x^2)^{1/5}$

Definition of $a^{\frac{m}{n}}$

If $a^{\frac{m}{n}}$ is a nonzero real number, then

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}$$

Example 5: Rewrite each of the following with a positive exponent. Simplify, if possible. Assume all variables represent nonnegative quantities.

a. $49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}} = \frac{1}{\sqrt{49}} = ?$

$= \frac{1}{7}$

b. $32^{-\frac{3}{5}} = \frac{1}{32^{3/5}} = \frac{1}{\left(\sqrt[5]{32}\right)^3} = \frac{1}{(2)^3} = \frac{1}{8}$

c. $(2ab)^{-\frac{2}{3}} = \frac{1}{(2ab)^{2/3}}$

d. $(-27)^{-\frac{2}{3}} = \frac{1}{(-27)^{2/3}} = \frac{1}{\left(\sqrt[3]{-27}\right)^2} = \frac{1}{(-3)^2} = \frac{1}{9}$

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49
7 7
$7^2 = 49$

32
4 8
2 2 2 4
2 2
$32 = 2^5$

$2^3 = 2 \cdot 2 \cdot 2 = 8$

27
3 9
3 3
$3^3 = 27$

$-27 = (-3)^3$

$(-3)^2 = (-3)(-3)$
$= 9$

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Properties of Rational Exponents

If m and n are rational exponents, and a and b are real numbers for which the following expressions are defined, then

1. $b^m \bullet b^n = b^{m+n}$
2. $\dfrac{b^m}{b^n} = b^{m-n}$
3. $(b^m)^n = b^{mn}$
4. $(ab)^n = a^n b^n$
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

spwk

16 → 4 4 → 2 2 2 2

$16 = 2\cdot2\cdot2\cdot2$
$= 2^4$

4 → 2 2

$4 = 2^2$

Example 6: Simplify the following expressions with rational exponents. Express all answers with positive exponents. Assume all variables represent nonnegative quantities.

a. $8^{\frac{2}{3}} \bullet 8^{\frac{4}{3}} = 8^{\frac{2}{3}+\frac{4}{3}} = 8^{\frac{6}{3}} = 8^2 = 8\cdot8 = 64$

b. $\dfrac{7^{\frac{5}{8}}}{7^{\frac{3}{8}}} = 7^{\frac{5}{8}-\frac{3}{8}} = 7^{\frac{2}{8}} = 7^{\frac{2\cdot1}{2\cdot4}} = 7^{\frac{1}{4}}$

c. $(3xy^2)^{\frac{5}{7}} = 3^{\frac{5}{7}} x^{\frac{5}{7}} (y^2)^{\frac{5}{7}} = 3^{\frac{5}{7}} x^{\frac{5}{7}} y^{\frac{2}{1}\cdot\frac{5}{7}} = 3^{\frac{5}{7}} x^{\frac{5}{7}} y^{\frac{10}{7}}$

d. $\dfrac{16x^{\frac{2}{3}}}{4x^{\frac{1}{2}}} = \dfrac{2\cdot2\cdot2\cdot2\ x^{\frac{2}{3}-\frac{1}{2}}}{2\cdot2} = 4x^{\frac{2}{3}\cdot\frac{2}{2}-\frac{1}{2}\cdot\frac{3}{3}} = 4x^{\frac{4}{6}-\frac{3}{6}} = \underline{4x^{\frac{1}{6}}}$

or $= \dfrac{2^4 \cdot x^{\frac{2}{3}}}{2^2 \cdot x^{\frac{1}{2}}} = 2^{4-2} x^{\frac{2}{3}-\frac{1}{2}} = 2^2 x^{\frac{4}{6}-\frac{3}{6}} = \underline{4x^{\frac{1}{6}}}$

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Simplifying Radical Expressions Using Rational Exponents

To simplify a radical expression by using rational exponents:
1. Rewrite each radical expression as an exponential expression with a rational exponent.
2. Simplify using properties of rational exponents.
3. Rewrite your answer in radical notation when rational exponents still appear.

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$8 = 2 \cdot 2 \cdot 2$
$8 = 2^3$

$16 = 2 \cdot 2 \cdot 2 \cdot 2$
$= 2^4$

$64 = 8 \cdot 8$
$64 = 2^3 \cdot 2^3$
$64 = 2^6$

$\frac{3}{6} = \frac{3 \cdot 1}{3 \cdot 2} = \frac{1}{2}$

$\frac{1}{2} + \frac{2}{3}$
$= \frac{1}{2} \cdot \frac{3}{3} + \frac{2}{3} \cdot$
$= \frac{3}{6} + \frac{4}{6}$
$= \frac{7}{6}$

Example 7: Use rational exponents to simplify. Assume all variables represent nonnegative quantities.

a. $\sqrt[3]{8x^2} = \left(8x^2\right)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left(x^2\right)^{\frac{1}{3}} = \sqrt[3]{8} \cdot x^{\frac{2}{1} \cdot \frac{1}{3}} = \underline{2x^{\frac{2}{3}}}$

b. $\sqrt{16xy^4} = (2^4 x y^4)^{\frac{1}{2}} = (2^4)^{\frac{1}{2}} x^{\frac{1}{2}} (y^4)^{\frac{1}{2}}$
$= 2^{\frac{4}{2}} x^{\frac{1}{2}} y^{4/2} = 2^2 x^{\frac{1}{2}} y^2 = \underline{4x^{\frac{1}{2}} y^2}$

c. $\sqrt[6]{64x^3} = (2^6 x^3)^{1/6} = (2^6)^{1/6} (x^3)^{1/6}$
$= 2^{6/6} x^{3/6} = \underline{2x^{\frac{1}{2}}}$

d. $\sqrt{x} \bullet \sqrt[3]{x^2} = x^{1/2} \cdot x^{2/3} = x^{\frac{1}{2} + \frac{2}{3}} = x^{\frac{3}{6} + \frac{4}{6}}$
$= \underline{x^{7/6}}$

e. $\sqrt{\sqrt[4]{x^3}} = ((x^3)^{\frac{1}{4}})^{\frac{1}{2}} = x^{\frac{3}{1} \cdot \frac{1}{4} \cdot \frac{1}{2}} = x^{\frac{3}{8}}$

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Application of Rational Exponents

Example 8: The function $f(x) = 70x^{\frac{3}{4}}$ models the number of calories per day, f(x), that a person needs to maintain life in terms of that person's weight, x, in kilograms. (1 kilogram is approximately 2.2 pounds.) Use the model and a calculator to find how many calories per day are required to maintain life for a person who weighs 55 kilograms (about 121 pounds). Round your answer to the nearest calorie.

x = person's weight in Kilograms

For x = 55, find f(55):

$f(55) = 70 \cdot (55)^{3/4}$

$f(55) \approx 70 \cdot (20.1963)$

$f(55) \approx 1{,}413.74$

$f(55) \approx 1{,}414$

ANS: For a person who weighs 55 kilograms, approximately 1,414 calories are needed to maintain life.

Example 9: Use your calculator to evaluate the following to three decimal places.

a. $(234)^{\frac{1}{4}} \approx 3.911$

b. $(-655)^{\frac{2}{3}} \approx 75.421$

c. $(45)^{\frac{3}{4}} + \sqrt[3]{47} \approx 17.3744 + 3.6088$

≈ 20.9832

≈ 20.983

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Answers Section 10.2

Example 1:
a. 6
b. –2
c. $\sqrt[5]{9xy^2}$
d. $|x|$

Example 2:
a. $(5xy)^{\frac{1}{4}}$
b. $(3xy^2)^{\frac{1}{3}}$
c. $(4a^2b)^{\frac{1}{5}}$
d. $(3xy)^{\frac{1}{2}}$

Example 3:
a. 64
b. 4
c. Not a real number
d. –8

Example 4:
a. $8^{\frac{5}{4}}$
b. $(3x)^{\frac{2}{3}}$
c. $(5xy)^{\frac{7}{6}}$
d. $(25x^2)^{\frac{1}{5}}$

Example 5:
a. $\frac{1}{7}$
b. $\frac{1}{8}$
c. $\frac{1}{(2ab)^{\frac{2}{3}}}$
d. $\frac{1}{9}$

Example 6:
a. 64
b. $7^{\frac{1}{4}}$
c. $3^{\frac{5}{7}}x^{\frac{5}{7}}y^{\frac{10}{7}}$
d. $4x^{\frac{1}{6}}$

Example 7:
a. $2x^{\frac{2}{3}}$
b. $4x^{\frac{1}{2}}y^2$
c. $2x^{\frac{1}{2}}$
d. $x^{\frac{7}{6}}$
e. $x^{\frac{3}{8}}$

Example 8:
a. x = 55 kg., f(55) ≅ 1414 calories

Example 9:
a. 3.911
b. 75.421
c. 20.983

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

10.3 Multiplying and Simplifying Radical Expressions

The Product Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

$$\sqrt[n]{a} \bullet \sqrt[n]{b} = \sqrt[n]{ab}.$$

The product of two nth roots is the nth root of the product.

Note that in order to multiply two radicals, the radicals must have the same index.

Example 1: Multiply. Assume all variables represent nonnegative quantities.

a. $\sqrt{5} \bullet \sqrt{6} = \sqrt{30}$

b. $\sqrt[3]{x} \bullet \sqrt[3]{3y} = \sqrt[3]{3xy}$

c. $\sqrt[5]{5x^2} \bullet \sqrt[5]{2x^2} = \sqrt[5]{5x^2 \cdot 2x^2}$
$= \sqrt[5]{10x^4}$

d. $\sqrt{2x+1} \bullet \sqrt{3x+7} = \sqrt{(2x+1)(3x+7)}$
$= \sqrt{(2x)(3x)+(2x)(7)+(1)(3x)+(1)(7)}$
$= \sqrt{6x^2+14x+3x+7}$
$= \sqrt{6x^2+17x+7}$

Simplifying Radical Expressions by Factoring

A radical expression whose index is n is said to be simplified when its radicand has no factors that are perfect nth powers.

To simplify a radical expression of index n:
1. Write the radicand in factored form, where all possible perfect nth powers appear as one of the factors.
2. Use the product rule to take the nth root of each factor.
3. Find the nth root of the perfect nth power.

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Example 2: Simplify. Assume all variables represent nonnegative quantities.

a. $\sqrt{18} = \sqrt{3^2 \bullet 2} = \sqrt{3^2} \bullet \sqrt{?} = ?$
$= \sqrt{3^2} \cdot \sqrt{2} = \underline{3\sqrt{2}}$

b. $\sqrt{72} = \sqrt{3^2 \cdot 2^2} \cdot \sqrt{2}$
$= 3 \cdot 2 \cdot \sqrt{2}$
$= \underline{6\sqrt{2}}$

c. $\sqrt[3]{108} = \sqrt[3]{3^3} \cdot \sqrt[3]{2^2}$
$= 3 \cdot \sqrt[3]{4}$
$= \underline{3\sqrt[3]{4}}$

d. $\sqrt[4]{162} = \sqrt[4]{3^4} \cdot \sqrt[4]{2}$
$= 3 \cdot \sqrt[4]{2}$
$= \underline{3\sqrt[4]{2}}$

e. $\sqrt{50x^3} = \sqrt{5^2 \cdot x^2} \cdot \sqrt{2x}$
$= 5 \cdot x \cdot \sqrt{2x}$
$= \underline{5x\sqrt{2x}}$

f. $\sqrt{27xy^5} = \sqrt{3^2 (y^2)^2} \cdot \sqrt{3xy}$
$= 3y^2\sqrt{3xy}$

g. $\sqrt[3]{2x^5y^4} = \sqrt[3]{x^3 \cdot y^3} \cdot \sqrt[3]{2x^2y}$
$= x \cdot y \cdot \sqrt[3]{2x^2y}$
$= xy\sqrt[3]{2x^2y}$

SDWK

$18 = 3^2 \cdot 2$

$72 = 3^2 \cdot 2^2 \cdot 2$

$108 = 3^3 \cdot 2^2$

$162 = 3^4 \cdot 2$

$50 = 5^2 \cdot 2$

$27 = 3^2 \cdot 3$

$y^5 = y^4 \cdot y$

$y^5 = (y^2)^2 \cdot y$

Using Absolute Values When Simplifying Even Roots: For the remainder of the chapter, assume that no radicands involve negative quantities raised to even powers. Thus, absolute value bars are not necessary when taking even roots. One exception is the process of simplifying a **radical function**. In this case, absolute value bars must be used when taking even roots of even powers.

Example 3: Simplify.

$f(x) = \sqrt{2x^2 + 12x + 18} = \sqrt{2(x^2 + 6x + 9)}$
$= \sqrt{2(x+3)(x+3)} = \sqrt{2(x+3)^2} = \sqrt{(x+3)^2} \cdot \sqrt{2}$
$f(x) = |x+3|\sqrt{2}$, or $f(x) = \sqrt{2}\,|x+3|$

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19/40

use $\sqrt{x^4} = \sqrt{(x^2)^2} = x^2$

& $\sqrt[4]{a^9} = \sqrt[4]{a^8}\cdot\sqrt[4]{a} = \sqrt[4]{(a^2)^4}\cdot\sqrt[4]{a} = a^2\sqrt[4]{a}$

Perfect nth Powers

When simplifying radicals of index n that involve variables raised to various powers, note that the perfect nth powers have exponents that are divisible by n.

Example 4: Simplify.

a. $\sqrt{3x^5y^3} = \sqrt{3x^4xy^2y} = \sqrt{3}\sqrt{x^4}\sqrt{x}\bullet\sqrt{y^2}\sqrt{y} = ?\ \sqrt{3}\cdot x^2\cdot\sqrt{x}\cdot y\cdot\sqrt{y} = x^2y\sqrt{3xy}$

or $= \sqrt{x^4\cdot y^2}\cdot\sqrt{3xy} = x^2\cdot y\cdot\sqrt{3xy} = x^2y\sqrt{3xy}$

b. $\sqrt{8x^4y^6} = \sqrt{2^2(x^2)^2(y^3)^2}\cdot\sqrt{2}$
$= 2\cdot x^2\cdot y^3\cdot\sqrt{2}$
$= 2x^2y^3\sqrt{2}$

c. $\sqrt[3]{3x^4y^3} = \sqrt[3]{x^3y^3}\cdot\sqrt[3]{3x}$
$= x\cdot y\cdot\sqrt[3]{3x}$
$= xy\sqrt[3]{3x}$

d. $\sqrt[3]{12a^5b^4} = \sqrt[3]{a^3\cdot b^3}\cdot\sqrt[3]{12a^2b}$
$= a\cdot b\cdot\sqrt[3]{12a^2b}$
$= ab\sqrt[3]{12a^2b}$

e. $\sqrt[4]{24a^9b^5} = \sqrt[4]{a^8\cdot b^4}\cdot\sqrt[4]{24ab}$
$= \sqrt[4]{(a^2)^4\cdot b^4}\cdot\sqrt[4]{24ab} = a^2b\cdot\sqrt[4]{24ab}$
$= a^2b\sqrt[4]{24ab}$

f. $\sqrt[5]{64x^{10}y^7} = \sqrt[5]{2^5x^{10}y^5}\cdot\sqrt[5]{2y^2}$
$= \sqrt[5]{2^5\cdot(x^2)^5y^5}\cdot\sqrt[5]{2y^2} = 2\cdot x^2\cdot y\cdot\sqrt[5]{2y^2}$
$= 2x^2y\sqrt[5]{2y^2}$

g. $\sqrt[6]{2x^{10}y^8} = \sqrt[6]{x^6y^6}\cdot\sqrt[6]{2x^4y^2}$
$= x\cdot y\sqrt[6]{2x^4y^2}$

h. $\sqrt[7]{128x^{10}y^5} = \sqrt[7]{2^7x^7}\cdot\sqrt[7]{x^3y^5}$
$= 2x\sqrt[7]{x^3y^5}$

SD work

$8 = 2\cdot2\cdot2$
$8 = 2^2\cdot2$

12 → 2, 6 → 2, 3
$12 = 2^2\cdot3$

24 → 2, 12 → 2, 6 → 2, 3
$24 = 2^3\cdot3$

64 → 8, 8 → 2, 4, 2, 4 → 2, 2, 2, 2
$64 = 2^6$
$64 = 2^5\cdot2$

128 → 2, 64
$128 = 2\cdot64$
$= 2\cdot2^6$
$= 2^7$

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Multiplying and Simplifying Radicals

To multiply radicals that have the same index, n:

- Use the product rule for nth roots to multiply the radicals, and
- Simplify the result by factoring and taking the nth root of the factors that are perfect nth powers.

Example 5: Multiply and simplify.

a. $\sqrt{12} \bullet \sqrt{6} = \sqrt{12 \cdot 6}$
$= \sqrt{2^2 \cdot 3^2} \cdot \sqrt{2} = 2 \cdot 3 \cdot \sqrt{2} = \underline{6\sqrt{2}}$

b. $\sqrt{2x} \bullet \sqrt{8x} = \sqrt{2x \cdot 8x}$
$= \sqrt{16x^2}$
$= \underline{4x}$

c. $3\sqrt{15} \bullet 5\sqrt{6} = 15 \cdot \sqrt{15 \cdot 6}$
$= 15 \cdot \sqrt{3^2} \cdot \sqrt{2 \cdot 5}$
$= 15 \cdot 3 \cdot \sqrt{10}$
$= \underline{45\sqrt{10}}$

d. $\sqrt{5xy} \bullet \sqrt{10xy^2} = \sqrt{5 \cdot 10 \cdot x^2 y^3}$
$= \sqrt{5^2 x^2 y^2} \cdot \sqrt{2 \cdot y}$
$= \underline{5xy\sqrt{2y}}$

e. $\sqrt[3]{4x^2} \bullet \sqrt[3]{4x} = \sqrt[3]{4 \cdot 4 \cdot x^3}$
$= \sqrt[3]{2^3 x^3} \cdot \sqrt[3]{2}$
$= \underline{2x\sqrt[3]{2}}$

f. $\sqrt[5]{8x^4y^3} \bullet \sqrt[5]{8xy^4} = \sqrt[5]{8 \cdot 8 \cdot x^5 \cdot y^7}$
$= \sqrt[5]{2^5 \cdot x^5 \cdot y^5} \cdot \sqrt[5]{2y^2}$
$= 2 \cdot x \cdot y \cdot \sqrt[5]{2y^2}$
$= \underline{2xy\sqrt[5]{2y^2}}$

SD WK

12
2 6
2 3
6
2 3
$12 \cdot 6 = 2^2 \cdot 3 \cdot 2$
$= 2^2 \cdot 3^2 \cdot 2$

15 6
3 5 2 3
$15 \cdot 6 = 3 \cdot 5 \cdot 3$
$= 3^2 \cdot 2 \cdot 5$

$5 \cdot 10$
$= 5 \cdot 5 \cdot 2$
$= 5^2 \cdot 2$

$4 \cdot 4$
$= 2 \cdot 2 \cdot 2 \cdot 2$
$= 2^3 \cdot 2$

$8 \cdot 8$
$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$= 2^5 \cdot 2$

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Answers Section 10.3

Example 1:
a. $\sqrt{30}$
b. $\sqrt[3]{3xy}$
c. $\sqrt[5]{10x^4}$
d. $\sqrt{6x^2+17x+7}$

Example 2:
a. $3\sqrt{2}$
b. $6\sqrt{2}$
c. $3\sqrt[3]{4}$
d. $3\sqrt[4]{2}$
e. $5x\sqrt{2x}$
f. $3y^2\sqrt{3xy}$
g. $xy\sqrt[3]{2x^2y}$

Example 3: $f(x) = \sqrt{2}|x+3|$, or $|x+3|\sqrt{2}$

Example 4:
a. $x^2y\sqrt{3xy}$
b. $2x^2y^3\sqrt{2}$
c. $xy\sqrt[3]{3x}$
d. $ab\sqrt[3]{12a^2b}$
e. $a^2b\sqrt[4]{24ab}$
f. $2x^2y\sqrt[5]{2y^2}$
g. $xy\sqrt[6]{2x^4y^2}$
h. $2x\sqrt[7]{x^3y^5}$

Example 5:
a. $6\sqrt{2}$
b. $4x$
c. $45\sqrt{10}$
d. $5xy\sqrt{2y}$
e. $2x\sqrt[3]{2}$
f. $2xy\sqrt[5]{2y^2}$

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22/40

10.4 Adding, Subtracting and Dividing Radical Expressions

Adding and Subtracting Radical Expressions

Radical expressions that have the same index and the same radicand are called "like radicals". Like radicals can be combined by adding or subtracting their coefficients.

Please note that in order to add (or subtract) two radicals, the two indicies **AND** the two radicands must be **identical**.

Example 1: Simplify.

a. $8\sqrt{5}-10\sqrt{5}=-2\sqrt{5}$

b. $3\sqrt[3]{3}+5\sqrt[3]{3}-4\sqrt{3} = 8\sqrt[3]{3}-4\sqrt{3}$

c. $x\sqrt{7}-2\sqrt{7} = (x-2)\sqrt{7}$

d. $\sqrt{15}+4\sqrt{15}-x\sqrt{15} = 5\sqrt{15}-x\sqrt{15}$
$= (5-x)\sqrt{15}$

In some cases radicals can be combined only after they have been simplified.

Example 2: Simplify.

a. $\sqrt{12}+\sqrt{27}=\sqrt{4*3}+\sqrt{9*3}=2\sqrt{3}+3\sqrt{3}=? = 5\sqrt{3}$

b. $\sqrt{20}+\sqrt{5} = \sqrt{2^2}\cdot\sqrt{5}+\sqrt{5} = 2\sqrt{5}+\sqrt{5} = 3\sqrt{5}$

c. $2\sqrt{18}+3\sqrt{8} = 2\sqrt{3^2}\cdot\sqrt{2}+3\sqrt{2^2}\sqrt{2}$
$= 2\cdot3\cdot\sqrt{2}+3\cdot2\cdot\sqrt{2}$
$= 6\sqrt{2}+6\sqrt{2}$
$= 12\sqrt{2}$

18
2 9
3 3
$18 = 3^2\cdot2$

Scratch work:

20
5 4
2 2
$20 = 2^2\cdot5$

12
2 6
2 3
$12 = 2^2\cdot3 = 4\cdot3$

27
3 9
3 3
$27 = 3^2\cdot3 = 9\cdot$

$8 = 2\cdot2\cdot2$
$= 2^2\cdot2$

d. $\sqrt{2x^2y} + \sqrt{18y} = \sqrt{x^2}\cdot\sqrt{2y} + \sqrt{3^2}\cdot\sqrt{2y}$

$= x\sqrt{2y} + 3\sqrt{2y}$

$= (x+3)\sqrt{2y}$

e. $\sqrt[3]{16x^4y^2} + \sqrt[3]{54xy^2} = \sqrt[3]{2^3x^3}\cdot\sqrt[3]{2xy^2} + \sqrt[3]{3^3}\cdot\sqrt[3]{2xy^2}$

$= 2\cdot x\cdot\sqrt[3]{2xy^2} + 3\cdot\sqrt[3]{2xy^2}$

$= (2x+3)\sqrt[3]{2xy^2}$

f. $\sqrt[4]{4x^5y^3} + \sqrt[4]{32xy^7} = \sqrt[4]{x^4}\cdot\sqrt[4]{4xy^3} + \sqrt[4]{2^4y^4}\cdot\sqrt[4]{2xy^3}$

$= x\sqrt[4]{4xy^3} + 2y\sqrt[4]{2xy^3}$

$= (x+2y)\sqrt[4]{4xy^3}$

SDWK

$18 = 3^2\cdot 2$

$16 = 2^3\cdot 2$

$54 = 3^3\cdot 2$

$32 = 2^4\cdot 2$

Dividing Rational Expressions

The Quotient Rule

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$$

The nth root of a quotient is the quotient of the nth roots.

The quotient rule may be used to simplify radical expressions or to divide radical expressions.

Example 3: Use the quotient rule to **simplify** the radicals.

a. $\sqrt{\frac{15}{25}} = \frac{\sqrt{15}}{\sqrt{25}} = \frac{\sqrt{15}}{5}$

b. $\sqrt{\frac{8}{49}} = \frac{\sqrt{8}}{\sqrt{49}} = \frac{\sqrt{2^2}\cdot\sqrt{2}}{7} = \frac{2\sqrt{2}}{7}$

c. $\sqrt{\frac{15x}{25y^2}} = \frac{\sqrt{15x}}{\sqrt{25y^2}} = \frac{\sqrt{15x}}{5y}$

SDWK

$8 = 2\cdot2\cdot2$

$8 = 2^2\cdot 2$

$49 = 7\cdot7 = 7^2$

$25 = 5\cdot5 = 5^2$

SDWK

$8 = 2 \cdot 2 \cdot 2$
$8 = 2^3$

d. $\sqrt[3]{\dfrac{x^4}{8y^3}} = \dfrac{\sqrt[3]{x^4}}{\sqrt[3]{8y^3}} = \dfrac{\sqrt[3]{x^3} \cdot \sqrt[3]{x}}{2y} = \dfrac{x\sqrt[3]{x}}{2y}$

e. $\sqrt[4]{\dfrac{13y^7}{x^8}} = \dfrac{\sqrt[4]{13y^7}}{\sqrt[4]{x^8}} = \dfrac{\sqrt[4]{y^4} \cdot \sqrt[4]{13y^3}}{\sqrt[4]{(x^2)^4}} = \dfrac{y\sqrt[4]{13y^3}}{x^2}$

The quotient rule may also be used to divide radicals.
Example 4: Simplify.

a. $\dfrac{\sqrt{200}}{\sqrt{10}} = \sqrt{\dfrac{200}{10}} = \sqrt{20} = \sqrt{4*5} = 2\sqrt{5}$

b. $\dfrac{\sqrt{50xy^2}}{\sqrt{2xy}} = \sqrt{\dfrac{50xy^2}{2xy}} = \sqrt{25y}$
$= \sqrt{25}\sqrt{y} = 5\sqrt{y}$

c. $\dfrac{\sqrt{x^5y^3}}{\sqrt{xy}} = \sqrt{\dfrac{x^5y^3}{xy}} = \sqrt{x^4y^2}$
$= x^2y$

d. $\dfrac{\sqrt[3]{250x^5y^3}}{\sqrt[3]{2x^3}} = \sqrt[3]{\dfrac{250x^5y^3}{2x^3}} = \sqrt[3]{125x^2y^3}$
$= \sqrt[3]{5^3y^3} \cdot \sqrt[3]{x^2} = 5y\sqrt[3]{x^2}$

SDWK

125
5 25
5 5
$125 = 5^3$

Answers Section 10.4

Example 1:

a. $-2\sqrt{5}$

b. $8\sqrt[3]{3}-4\sqrt{3}$

c. $(x-2)\sqrt{7}$

d. $(5-x)\sqrt{15}$

Example 2:

a. $5\sqrt{3}$

b. $3\sqrt{5}$

c. $12\sqrt{2}$

d. $(x+3)\sqrt{2y}$

e. $(2x+3)\sqrt[3]{2xy^2}$

f. $x\sqrt[4]{4xy^3}+2y\sqrt[4]{2xy^3}$

Example 3:

a. $\dfrac{\sqrt{15}}{5}$

b. $\dfrac{2\sqrt{2}}{7}$

c. $\dfrac{\sqrt{15x}}{5y}$

d. $\dfrac{x\sqrt[3]{x}}{2y}$

e. $\dfrac{y\sqrt[4]{13y^3}}{x^2}$

Example 4:

a. $2\sqrt{5}$

b. $5\sqrt{y}$

c. x^2y

d. $5y\sqrt[3]{x^2}$

10.5 Multiplying with More Than One Term and Rationalizing Denominators

Multiplying Radicals with More Than One Term

$A\cdot(B+C)=A\cdot B+A\cdot C$

To multiply radicals with more than one term, use the distributive law. If the two expressions are both binomials, you may use the FOIL method.

Example 1: Simplify.

a. $\sqrt{7}\left(x+\sqrt{10}\right)=\sqrt{7}*x+\sqrt{7}*\sqrt{10}=$ OR $\sqrt{7}x+\sqrt{70}$
$=x\sqrt{7}+\sqrt{70}$

b. $\sqrt[3]{x}\left(\sqrt[3]{x}+\sqrt[3]{14}\right)=\sqrt[3]{x}\cdot\sqrt[3]{x}+\sqrt[3]{x}\cdot\sqrt[3]{14}=\sqrt[3]{x^2}+\sqrt[3]{14x}$

SAME oops!

SDWK
$\sqrt{7}\cdot\sqrt{7}=\sqrt{7^2}$
$=7$
$\sqrt{2}\cdot\sqrt{2}=\sqrt{2^2}$
$=2$

c. $\left(5+\sqrt{7}\right)\left(3-4\sqrt{7}\right)=5*3-5*4\sqrt{7}+\sqrt{7}*3-\sqrt{7}*4\sqrt{7}=?$
$=15-20\sqrt{7}+3\sqrt{7}-4\cdot7$
$=15-17\sqrt{7}-28$
$=-13-17\sqrt{7}$

d. $\left(5\sqrt{2}+\sqrt{7}\right)\left(3\sqrt{2}-4\sqrt{7}\right)$
$=(5\sqrt{2})(3\sqrt{2})+(5\sqrt{2})(-4\sqrt{7})+(\sqrt{7})(3\sqrt{2})+(\sqrt{7})(-4\sqrt{7})$
$=15\cdot2-20\sqrt{14}+3\sqrt{14}-4\cdot7$
$=30-17\sqrt{14}-28=2-17\sqrt{14}$

e. $\left(5+\sqrt{7}\right)^2$
$=(5+\sqrt{7})(5+\sqrt{7})$
$=(5)(5)+(5)(\sqrt{7})+(\sqrt{7})(5)+(\sqrt{7})(\sqrt{7})$
$=25+5\sqrt{7}+5\sqrt{7}+7=32+10\sqrt{7}$

f. $\left(5+\sqrt{7}\right)\left(3-4\sqrt{7}\right)$
$=(5)(3)+(5)(-4\sqrt{7})+(\sqrt{7})(3)+(\sqrt{7})(-4\sqrt{7})$
$=15-20\sqrt{7}+3\sqrt{7}-4\cdot7=15-17\sqrt{7}-28=-13-17\sqrt{7}$

g. $\left(5-\sqrt{7}\right)\left(5+\sqrt{7}\right)$
$=(5)(5)+(5)(\sqrt{7})+(-\sqrt{7})(5)+(-\sqrt{7})(\sqrt{7})$
$=25+5\sqrt{7}-5\sqrt{7}-7$
$=18$

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Rationalizing Denominators Containing One Term

To rationalize the denominator of a radical expression, you must rewrite the expression as an equivalent expression that does not contain any radicals in the denominator. When the denominator contains a single radical with an nth root, multiply the numerator and the denominator by a radical of index n that produces a perfect nth power in the denominator's radicand.

Example 2: Rationalize each denominator.

a. $\frac{\sqrt{10}}{\sqrt{3}} = \frac{\sqrt{10}}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{10 \cdot 3}}{\sqrt{3^2}} = \frac{\sqrt{30}}{3}$

b. $\frac{\sqrt{3}}{\sqrt{x}} = \frac{\sqrt{3}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{3 \cdot x}}{\sqrt{x^2}} = \frac{\sqrt{3x}}{x}$

c. $\sqrt{\frac{12}{5x}} = \frac{\sqrt{12}}{\sqrt{5x}} = \frac{\sqrt{2^2} \cdot \sqrt{3}}{\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}} = \frac{2 \cdot \sqrt{3 \cdot 5x}}{\sqrt{5^2 x^2}} = \frac{2\sqrt{15x}}{5x}$

d. $\frac{\sqrt[3]{14}}{\sqrt[3]{3}} = \frac{\sqrt[3]{14}}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \frac{\sqrt[3]{14 \cdot 9}}{\sqrt[3]{3^3}} = \frac{\sqrt[3]{126}}{3}$

e. $\frac{\sqrt[3]{6}}{\sqrt[3]{7x^2}} = \frac{\sqrt[3]{6}}{\sqrt[3]{7x^2}} * \frac{\sqrt[3]{7^2 x}}{\sqrt[3]{7^2 x}} = \frac{?}{\sqrt[3]{7^3 x^3}} = \frac{\sqrt[3]{6 \cdot 49 \cdot x}}{7x} = \frac{\sqrt[3]{294x}}{7x}$

$\sqrt[3]{6 \cdot 7^2 \cdot x}$

f. $\frac{\sqrt[3]{3x}}{\sqrt[3]{y^2}} = \frac{\sqrt[3]{3x}}{\sqrt[3]{y^2}} \cdot \frac{\sqrt[3]{y}}{\sqrt[3]{y}} = \frac{\sqrt[3]{3xy}}{\sqrt[3]{y^3}} = \frac{\sqrt[3]{3xy}}{y}$

g. $\frac{\sqrt[4]{5xy}}{\sqrt[4]{2z^2}} = \frac{\sqrt[4]{5xy} \cdot \sqrt[4]{2^3 \cdot z^2}}{\sqrt[4]{2z^2} \cdot \sqrt[4]{2^3 \cdot z^2}} = \frac{\sqrt[4]{5 \cdot 8xyz^2}}{\sqrt[4]{2^4 z^4}} = \frac{\sqrt[4]{40xyz^2}}{2z}$

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$a+b$ & $a-b$

use $(a+b)\cdot(a-b) = a^2-b^2$

Rationalizing Denominators Containing Two Terms
Radical expressions that involve the sum and the difference of the same two terms are called conjugates.
Example 3: Find the conjugate of each expression.

a. $\sqrt{2}+3$; $\sqrt{2}-3$

b. $5-3\sqrt{5}$; $5+3\sqrt{5}$

c. $\sqrt[3]{x}-3$; $\sqrt[3]{x}+3$

To rationalize a denominator that contains two terms, multiply both numerator and denominator by the conjugate of the denominator.
Example 4: Rationalize each denominator.

a. $\dfrac{15}{\sqrt{6}+1} = \left[\dfrac{15}{\sqrt{6}+1}\right]\cdot\left[\dfrac{\sqrt{6}-1}{\sqrt{6}-1}\right] = \dfrac{15[\sqrt{6}-1]}{(\sqrt{6})^2-(1)^2} = \dfrac{15\cdot[\sqrt{6}-1]}{6-1}$

$= \dfrac{3\cdot5\cdot[\sqrt{6}-1]}{1\cdot5} = 3[\sqrt{6}-1] = 3\sqrt{6}-3$

b. $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{7}-1} = \left[\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{7}-1}\right]\cdot\left[\dfrac{\sqrt{7}+1}{\sqrt{7}+1}\right] = \dfrac{(\sqrt{5})(\sqrt{7})+(\sqrt{5})(1)+(\sqrt{3})(\sqrt{7})+(\sqrt{3})(1)}{(\sqrt{7})^2-(1)^2}$

$= \dfrac{\sqrt{35}+\sqrt{5}+\sqrt{21}+\sqrt{3}}{7-1} = \dfrac{\sqrt{35}+\sqrt{5}+\sqrt{21}+\sqrt{3}}{6}$

c. $\dfrac{\sqrt{x}+2}{\sqrt{x}-1} = \left[\dfrac{\sqrt{x}+2}{\sqrt{x}-1}\right]\cdot\left[\dfrac{\sqrt{x}+1}{\sqrt{x}+1}\right] = \dfrac{(\sqrt{x})^2+(\sqrt{x})(1)+(2)(\sqrt{x})+(2)(1)}{(\sqrt{x})^2-(1)^2}$

$= \dfrac{x+\sqrt{x}+2\sqrt{x}+2}{x-1} = \dfrac{x+3\sqrt{x}+2}{x-1}$

d. $\dfrac{\sqrt{3}-\sqrt{7}}{\sqrt{6}+\sqrt{2}} = \left[\dfrac{\sqrt{3}-\sqrt{7}}{\sqrt{6}+\sqrt{2}}\right]\cdot\left[\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right] = \dfrac{(\sqrt{3})(\sqrt{6})+(\sqrt{3})(-\sqrt{2})+(-\sqrt{7})(\sqrt{6})+(-\sqrt{7})(-\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}$

$= \dfrac{\sqrt{3^2}\sqrt{2}-\sqrt{6}-\sqrt{42}+\sqrt{14}}{6-2} = \dfrac{3\sqrt{2}-\sqrt{6}-\sqrt{42}+\sqrt{14}}{4}$

or $\dfrac{3\sqrt{2}-\sqrt{42}-\sqrt{6}+\sqrt{14}}{4}$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

29/40

Answers Section 10.5

Example 1:

a. $\sqrt{7x}+\sqrt{70}$, or $x\sqrt{7}+\sqrt{70}$

b. $\sqrt[3]{x^2}+\sqrt[3]{14x}$

c. $-13-17\sqrt{7}$

d. $2-17\sqrt{14}$

e. $32+10\sqrt{7}$

f. $-13-17\sqrt{7}$

g. 18

Example 2:

a. $\frac{\sqrt{30}}{3}$

b. $\frac{\sqrt{3x}}{x}$

c. $\frac{2\sqrt{15x}}{5x}$

d. $\frac{\sqrt[3]{126}}{3}$

e. $\frac{\sqrt[3]{294x}}{7x}$

f. $\frac{\sqrt[3]{3xy}}{y}$

g. $\frac{\sqrt[4]{40xyz^2}}{2z}$

Example 3:

a. $\sqrt{2}-3$

b. $5+3\sqrt{5}$

c. $\sqrt[3]{x}+3$

Example 4:

a. $3(\sqrt{6}-1)$, or $3\sqrt{6}-3$

b. $\frac{\sqrt{35}+\sqrt{21}+\sqrt{5}+\sqrt{3}}{6}$

c. $\frac{x+3\sqrt{x}+2}{x-1}$

d. $\frac{\sqrt{18}-\sqrt{42}-\sqrt{6}+\sqrt{14}}{4}$, or $\frac{3\sqrt{2}-\sqrt{42}-\sqrt{6}+\sqrt{14}}{4}$, or $\frac{3\sqrt{2}-\sqrt{6}-\sqrt{42}+\sqrt{14}}{4}$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

10.6 Radical Equations

Solving Radical Equations

In order to solve an equation that contains one or more nth roots,
1. If necessary, arrange the terms so that one radical is isolated on one side of the equation.
2. Raise both sides of the equation to the nth power to eliminate the nth root.
3. Solve the resulting equation. If this equation still contains nth roots, repeat steps l and 2 until all nth roots have been eliminated.
4. Test each proposed solution in the original equation to determine if it is a solution.

Example 1: Solve each radical equation. Note that each equation contains only one square root.

a. $\sqrt{5x-1}=8$

$(\sqrt{5x-1})^2=(8)^2$

$5x-1=64$

$1+5x-1=1+64$

$5x=65$

$\frac{5x}{5}=\frac{65}{5}$

$x=13$

check:

$\sqrt{5\cdot(13)-1}=8$

$\sqrt{65-1}=8$

$\sqrt{64}=8$

$8=8$

TRUE!

Solution Set = $\{13\}$

b. $\sqrt{4x-3}=5$

$(\sqrt{4x-3})^2=(5)^2$

$4x-3=25$

$3+4x-3=3+25$

$4x=28$

$\frac{4x}{4}=\frac{28}{4}$

$x=7$

check:

$\sqrt{4(7)-3}=5$

$\sqrt{28-3}=5$

$\sqrt{25}=5$

$5=5$

TRUE!

Solution Set = $\{7\}$

c. $\sqrt{5x-4}+2=6$

$-2+\sqrt{5x-4}+2=-2+6$

$\sqrt{5x-4}=4$

$(\sqrt{5x-4})^2=(4)^2$

$5x-4=16$

$4+5x-4=4+16$

$5x=20$

$\frac{5x}{5}=\frac{20}{5}$

$x=4$

check:

$\sqrt{5(4)-4}+2=6$

$\sqrt{20-4}+2=6$

$\sqrt{16}+2=6$

$4+2=6$

$6=6$

TRUE!

Solution Set = $\{4\}$

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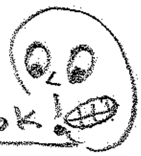
d. $\sqrt{2x+5}+11=6$

$\sqrt{2x+5}=-5$ ← Look!



$2x+5=(-5)^2$

$2x+5=25$

$-5+2x+5=-5+25$

$2x=20$

$\frac{2x}{2}=\frac{20}{2}$

$x=10$

check!

$\sqrt{2(10)+5}+11=6$

$\sqrt{20+5}+11=6$

$\sqrt{25}+11=6$

$5+11=6$

$16=6$

False!

Solution Set is $\varnothing$. No Solution!

x = ? Check this proposed solution to see if it works.

What is the solution, if any?

Solution Set: {7

e. $x=\sqrt{6x+7}$

$(x)^2=(\sqrt{6x+7})^2$

$x^2=6x+7$

$-6x-7+x^2=-6x-7+6x+7$

$x^2-6x-7=0$

$(x-7)(x+1)=0$

either

$x-7=0$, or $x+1=0$

$x=7$, or $x=-1$

check!	check!
$(7)=\sqrt{6(7)+7}$	$(-1)=\sqrt{6(-1)+7}$
$7=\sqrt{42+7}$	$-1=\sqrt{-6+7}$
$7=\sqrt{49}$	$-1=\sqrt{1}$
$7=7$	$-1=1$
TRUE!	False!

f. $x=\sqrt{3x+7}-3$

$x+3=3+\sqrt{3x+7}-3$

$(x+3)^2=(\sqrt{3x+7})^2$

$(x+3)(x+3)=3x+7$

$x^2+3x+3x+9=3x+7$

$x^2+6x+9=3x+7$

$-3x-7+x^2+6x+9=-3x-7+3x+7$

$x^2+3x+2=0$

$(x+2)(x+1)=0$

Either

$x+2=0$, or $x+1=0$

$x=-2$, or $x=-1$

check!	check:
$(-2)=\sqrt{3(-2)+7}-3$	$(-1)=\sqrt{3(-1)+7}-3$
$-2=\sqrt{-6+7}-3$	$-1=\sqrt{-3+7}-3$
$-2=\sqrt{1}-3$	$-1=\sqrt{4}-3$
$-2=1-3$	$-1=2-3$
$-2=-2$	$-1=-1$
TRUE!	TRUE!

Solution Set = {-2, -1

The following equations contain roots other than square roots. Remember that to eliminate an nth root, isolate the term containing the root, and then raise both sides to the nth power.

Example 2: Solve each equation.

a. $\sqrt[3]{x-1}=3$

$(\sqrt[3]{x-1})^3=(3^3)$ (Cube both sides)

$x-1=27$ (Solve this equation)

$x=28$

Answer : {28}

b. $\sqrt[3]{5x-1}=4$

$(\sqrt[3]{5x-1})^3=(4)^3$

$5x-1=64$

$5x-1+1=64+1$

$5x=65$

$\frac{5x}{5}=\frac{65}{5}$

$x=13$

check:

$\sqrt[3]{5(13)-1}=4$

$\sqrt[3]{65-1}=4$

$\sqrt[3]{64}=4$

$4=4$

TRUE!

Solution Set = {13}

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer

c. $\sqrt[4]{7x+2}+15=17$ (Hint: Isolate the fourth root first).

$-15+\sqrt[4]{7x+2}+15=-15+17$
$\sqrt[4]{7x+2}=2$
$(\sqrt[4]{7x+2})^4=(2)^4$
$7x+2=16$
$-2+7x+2=-2+16$
$7x=14$
$\frac{7x}{7}=\frac{14}{7}$
$x=2$

check:
$\sqrt[4]{7(2)+2}+15=17$
$\sqrt[4]{14+2}+15=17$
$\sqrt[4]{16}+15=17$
$2+15=17$
$17=17$
TRUE!

$\boxed{\text{Solution Set} = \{2\}}$

d. $(x-3)^{\frac{1}{3}}=5$ (Hint: The $\frac{1}{3}$ power means cube root.)

$[(x-3)^{\frac{1}{3}}]^3=(5)^3$
$(x-3)^{3/3}=125$
$x-3=125$
$x-3+3=125+3$
$x=128$

check:
$[(128)-3]^{\frac{1}{3}}=5$
$(125)^{\frac{1}{3}}=5$
$\sqrt[3]{125}=5$
$5=5$
TRUE!

$\boxed{\text{Solution Set} = \{128\}}$

Some radical equations contain more than one nth root. To solve these equations, radical terms must be isolated one-by-one and eliminated by raising both sides to the nth power.

Example 3: Solve each equation.

a. $\sqrt{x-8}=\sqrt{x}-2$ Isolate one radical.

$\left(\sqrt{x-8}\right)^2=\left(\sqrt{x}-2\right)^2$ Square both sides.

$\boxed{(\sqrt{x}-2)(\sqrt{x}-2)}$

$x-8=\left(\sqrt{x}\right)^2-4\sqrt{x}+4$ Left side: $\left(\sqrt{a}\right)^2=a$, Right side: FOIL

$x-8=x+4-4\sqrt{x}$ Simplify.

$-12=-4\sqrt{x}$ Isolate the remaining radical.

$3=\sqrt{x}$ Simplify.

$9=x$ Square both sides.

Check this proposed solution: {9}. Does it work?

Check!
$\sqrt{(9)-8}=\sqrt{(9)}-2$
$\sqrt{1}=3-2$
$1=1$
TRUE!

$\boxed{\text{Solution Set} = \{9\}}$

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b. $\sqrt{x-4}+\sqrt{x+4}=4$

$-\sqrt{x+4}+\sqrt{x-4}+\sqrt{x+4}=-\sqrt{x+4}+4$

$\sqrt{x-4}=-\sqrt{x+4}+4$

$(\sqrt{x-4})^2=(-\sqrt{x+4}+4)^2$

$x-4=(\sqrt{x+4})^2-4\sqrt{x+4}-4\sqrt{x+4}+(4)^2$

$x-4=x+4-8\sqrt{x+4}+16$

$x-4=x+20-8\sqrt{x+4}$

$-x-20+x-4=-x-20+x+20-8\sqrt{x+4}$

$-24=-8\sqrt{x+4}$

$\frac{-24}{-8}=\frac{-8\sqrt{x+4}}{-8}$

$3=\sqrt{x+4}$

$(3)^2=(\sqrt{x+4})^2$

$9=x+4$

$-4+9=-4+x+4$

$\underline{5=x}$

scrwk

$(-\sqrt{x+4}+4)^2$

$=(-\sqrt{x+4}+4)(-\sqrt{x+4}+4)$

$=(\sqrt{x+4})^2-4\sqrt{x+4}-4\sqrt{x+4}+16$

$=x+4-8\sqrt{x+4}+16$

$=x+20-8\sqrt{x+4}$

Check!

$\sqrt{(5)-4}+\sqrt{(5)+4}=4$

$\sqrt{1}+\sqrt{9}=4$

$1+3=4$

$4=4$

TRUE!

Solution Set $=\{5\}$

c. $\sqrt{x-4}+\sqrt{x+1}=5$

$-\sqrt{x+1}+\sqrt{x-4}+\sqrt{x+1}=-\sqrt{x+1}+5$

$\sqrt{x-4}=-\sqrt{x+1}+5$

$(\sqrt{x-4})^2=(-\sqrt{x+1}+5)^2$

$x-4=(\sqrt{x+1})^2-5\sqrt{x+1}-5\sqrt{x+1}+(5)^2$

$x-4=x+1-10\sqrt{x+1}+25$

$x-4=x+26-10\sqrt{x+1}$

$-x-26+x-4=-x-26+x+26-10\sqrt{x+1}$

$-30=-10\sqrt{x+1}$

$\frac{-30}{-10}=\frac{-10\sqrt{x+1}}{-10}$

$3=\sqrt{x+1}$

$(3)^2=(\sqrt{x+1})^2$

$9=x+1$

$-1+9=-1+x+1$

$\underline{8=x}$

check:

$\sqrt{(8)-4}+\sqrt{(8)+1}=5$

$\sqrt{4}+\sqrt{9}=5$

$2+3=5$

$5=5$

TRUE!

Solution Set $=\{8\}$

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Applications of Square Root Functions

Example 4: Out of a group of 50,000 births, the number of people, f(x) surviving to age x is modeled by the function

$$f(x) = 5000\sqrt{100 - x}.$$

a. How many people in the group are expected to survive to age 80?

b. At what age are 35,000 people in the group still surviving?

(a). $x = 80$.
find $f(80)$ = number of people surviving to age 80

$f(80) = 5{,}000\sqrt{100 - (80)}$
$f(80) = 5{,}000 \cdot \sqrt{20}$
$f(80) = 5{,}000 \cdot \sqrt{4} \cdot \sqrt{5}$
$f(80) = 5{,}000 \cdot 2 \cdot \sqrt{5}$
$f(80) = 10{,}000\sqrt{5}$
$f(80) \approx 10{,}000 \cdot (2.23607\ldots)$
$f(80) \approx 22{,}360.7$
$f(80) \approx 22{,}360$

(b) Find x when $f(x) = 35{,}000$

$35{,}000 = f(x)$
$35{,}000 = 5{,}000\sqrt{100 - x}$
$\frac{35{,}000}{5{,}000} = \frac{5{,}000\sqrt{100-x}}{5{,}000}$
$7 = \sqrt{100 - x}$
$(7)^2 = (\sqrt{100 - x})^2$
$49 = 100 - x$
$-100 + 49 = -100 + 100 - x$
$-51 = -x$
$-51 \cdot (-1) = (-x) \cdot (-1)$
$51 = x$

check:

$35{,}000 = 5{,}000\sqrt{100 - (51)}$
$35{,}000 = 5{,}000\sqrt{49}$
$35{,}000 = 5{,}000 \cdot (7)$
$35{,}000 = 35{,}000$
TRUE!

ANS
(a) Approximately 22,360 people should survive to age 80.

(b) We expect 35,000 people to survive to age 51.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer

Answers Section 10.6

Example 1:
a. {13}
b. { 7}
c. { 4}
d. No solution, or ∅
e. { 7} (Note that the proposed solution $x = -1$ does not work)
f. { −2, −1}

Example 2:
a. { 28}
b. {13}
c. { 2}
d. { 128}

Example 3:
a. { 9}
b. { 5}
c. { 8}

Example 4:
a. $f(x) = 5000\sqrt{20} \cong 22{,}360$ About 22,360 people are expected to survive to age 80.
b. At age 51 there are still 35,000 people in the group surviving.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer

10.7 Complex Numbers

The Imaginary Unit i

The imaginary unit i is defined as
$$i=\sqrt{-1} \text{ where } i^2=-1.$$

If b is a positive number, then
$$\sqrt{-b}=\sqrt{b(-1)}=\sqrt{b}\sqrt{-1}=i\sqrt{b}$$

Example 1: Write each square root of a negative number as a multiple of i.

a. $\sqrt{-5}=\sqrt{-1*5}=\sqrt{-1}*\sqrt{5}=i\sqrt{5}$ or $\sqrt{5}\,i$

b. $\sqrt{-25} = \sqrt{-1\cdot 25} = \sqrt{-1}\cdot\sqrt{25} = i\cdot 5 = 5i$

c. $\sqrt{-16} = \sqrt{-1\cdot 16} = \sqrt{-1}\cdot\sqrt{16} = i\cdot 4 = 4i$

Complex Numbers and Imaginary Numbers

The set of all numbers in the form
$$a + bi$$
with real numbers a and b, and i, the imaginary unit, is called the set of complex numbers. The real number a is called the real part, and the real number b is called the imaginary part, of the complex number a + bi. Complex numbers can be further described as either:

- Imaginary, if a=0
- Real, if b = 0,
- Complex but not real, if neither a nor b is zero.

Example 2:

a. Consider the complex number 8 + 5i. What is the imaginary part of the number? The real part? How can you further describe this number?

b. Consider the complex number 8. What is the imaginary part of this complex number? How can you further describe this number?

c. Consider the complex number 5i. What is the imaginary part of this number? The real part? How can you further describe this number?

a. { • 5 • 8 • Complex, but not Real

b. { • zero • Real

c. { • 5 • zero • Imaginar

Note: Portions of this document are excerpted from the textbook *Algebra: A Combined Approach* by Elayn Martin-Gay.

$37/40$

Adding and Subtracting Complex Numbers

To add or subtract complex numbers:

1. (a + bi) + (c + di) = (a + c) + (b + d)i
   In words, add complex numbers by adding the real parts, adding the imaginary parts, and expressing the result as a complex number.
2. (a + bi) – (c + di) = (a – c) + (b – d)i
   In words, subtract complex numbers by subtracting the real parts, subtracting the imaginary parts, and expressing the result as a complex number.

Example 3: Simplify the following.

a. $(2-3i)+(5+7i) = (2+5)+(-3i+7i)$
$= 7+4i$

b. $(10+8i)-(2-6i) = 10+8i-2+6i$
$= 8+14i$

Multiplying Complex Numbers

To multiply complex numbers, use the distributive law and the FOIL method.

Example 4: Simplify the following.

a. $(2-3i)(5+7i) = (2)(5)+(2)(7i)+(-3i)(5)+(-3i)(7i)$
$= 10+14i-15i-21i^2$
$= 10-i-21(-1)$
$= 10-i+21 = 31-i$

b. $7i(3-i) = (7i)(3)+(7i)(-i)$
$= 21i-7i^2$
$= 21i-7(-1)$
$= 21i+7$
$= 7+21i$

The product rule for radicals only applies to real numbers. If a radical does not represent a real number, you must write the radical as a multiple of i before you use the product rule.

Example 5: Simplify the following.

a. $\sqrt{-4}\sqrt{-9} = 2i * 3i = ?$
$= 6i^2 = 6(-1) = -6$

b. $\sqrt{-5}\sqrt{-6} = \sqrt{-1}\cdot\sqrt{5}\cdot\sqrt{-1}\cdot\sqrt{6}$
$= i\sqrt{5}\cdot i\sqrt{6}$
$= i^2\cdot\sqrt{30} = (-1)\cdot\sqrt{30} = -\sqrt{30}$

Note: Portions of this document are excerpted from the textbook *Algebra: A Combined Approach* by Elayn Martin-Gay.

$(a+bi)\cdot(a-bi) = a^2 - abi + abi - b^2i^2$
$= a^2 - b^2(-1) = \underline{a^2 + b^2}$ ← "sum of squares"

Conjugates and Division of Complex Numbers

The conjugate of the complex number a + bi is the complex number a – bi. When a complex number is multiplied by its conjugate, the result is a real number.

Example 6: Multiply each complex number by its conjugate.

a. 7i ; $(7i)(-7i) = -49i^2 = -49\cdot(-1) = \underline{49}$

b. 3+7i ; $(3+7i)(3-7i) = (3)^2 - (3)(7i) + (3)(7i) - (7i)^2$
$= 9 - 21i + 21i - 49i^2$ $= 9 - 49\cdot(-1)$
$= 9 + 49 = \underline{58}$

c. 6+5i ; $(6+5i)\cdot(6-5i) = (6)^2 - (6)(5i) + (6)(5i) - (5i)^2$
$= 36 - 30i + 30i - 25i^2$ $= 36 - 25\cdot(-1)$
$= 36 + 25$
$= \underline{61}$

sum of squares

To divide two complex numbers, write in fraction form and then multiply the numerator and the denominator by the conjugate of the denominator.

Example 7: Simplify.

a. $\frac{3-i}{7i} = \left(\frac{3-i}{7i}\right)\cdot\left(\frac{-7i}{-7i}\right) = \frac{-21i+7i^2}{-49i^2} = \frac{-21i+7\cdot(-1)}{-49\cdot(-1)} = \frac{-21i-7}{49} = \frac{-7}{49} - \frac{21}{49}i$
$= \underline{\frac{-1}{7} - \frac{3}{7}i}$

b. $\frac{3-2i}{6+5i} = \left(\frac{3-2i}{6+5i}\right)\cdot\left(\frac{6-5i}{6-5i}\right) = ?\ \frac{18-15i-12i+10i^2}{36+25} = \frac{18-27i+10\cdot(-1)}{61} = \frac{18-27i-10}{61}$

sum of squares

$= \frac{8-27i}{61}$
$= \underline{\frac{8}{61} - \frac{27}{61}i}$

c. $\frac{3-i}{7-i} = \left(\frac{3-i}{7-i}\right)\cdot\left(\frac{7+i}{7+i}\right) = \frac{21+3i-7i-i^2}{49+1}$

sum of squares

$= \frac{21-4i-(-1)}{50}$
$= \frac{21-4i+1}{50}$
$= \frac{22-4i}{50}$

$= \frac{22}{50} - \frac{4}{50}i$
$= \frac{2\cdot 11}{2\cdot 25} - \frac{2\cdot 2}{2\cdot 25}i$
$= \underline{\frac{11}{25} - \frac{2}{25}i}$

Note: Portions of this document are excerpted from the textbook *Algebra: A Combined Approach* by Elayn Martin-Gay.

Powers of i

The powers of i cycle through four values: i, –1, –i, and 1.

Example 8: Simplify.

a. $i^1 = i$

b. $i^2 = -1$

c. $i^3 = i^2 \cdot i = -1 \cdot i = -i$

d. $i^4 = i^2 \cdot i^2 = (-1)\cdot(-1) = 1$

e. $i^5 = i^4 \cdot i = 1 \cdot i = i$

f. $i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$

g. $i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i$

h. $i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$

Simplifying Powers of i

To simplify a power of i 1. Express the given power of i in terms of i^2. 2. Replace i^2 by –1 and simplify. Use the fact that –1 to an even power is 1 and –1 to an odd power is –1.

Example 9: Simplify.

a. $i^{12} = (i^2)^6 = (-1)^6 = 1$

b. $i^{31} = (i^2)^{15} \bullet i = (-1)^{15} \bullet i = -1 \bullet i = -i$

c. $i^{52} = (i^2)^{26} = (-1)^{26} = 1$

d. $i^{79} = i^{78} \cdot i = (i^2)^{39} \cdot i = (-1)^{39} \cdot i = -1 \cdot i = -i$

Note: Portions of this document are excerpted from the textbook *Algebra: A Combined Approach* by Elayn Martin-Gay.

40/40

Answers Section 10.7

Example 1:

a. $i\sqrt{5}$

b. 5i

c. 4i

Example 2:

a. 8 is the real part and 5 is the imaginary part. The number is a complex number that is a "complex number that is not real".

b. 8 is the real part and 0 is the imaginary part. The number is a complex number that is "real".

c. 0 is the real part and 5 is the imaginary part. The number is a complex number that is "imaginary".

Example 3:

a. 7 + 4i

b. 8 + 14i

Example 4:

a. 31 – i

b. 7 + 21i

Example 5:

a. –6

b. $-\sqrt{30}$

Example 6:

a. 49

b. 58

c. 61

Example 7:

a. $\frac{-21i-7}{49}$, or $\frac{-1}{7} - \frac{3}{7}i$

b. $\frac{8-27i}{61}$, or $\frac{8}{61} - \frac{27}{61}i$

c. $\frac{22-4i}{50}$, or $\frac{11}{25} - \frac{2}{25}i$

Example 8:

a. i

b. –1

c. –i

d. 1

e. i

f. –I

g. –i

h. 1

Example 9:

a. 1

b. –i

c. 1

d. –i

Note: Portions of this document are excerpted from the textbook *Algebra: A Combined Approach* by Elayn Martin-Gay.